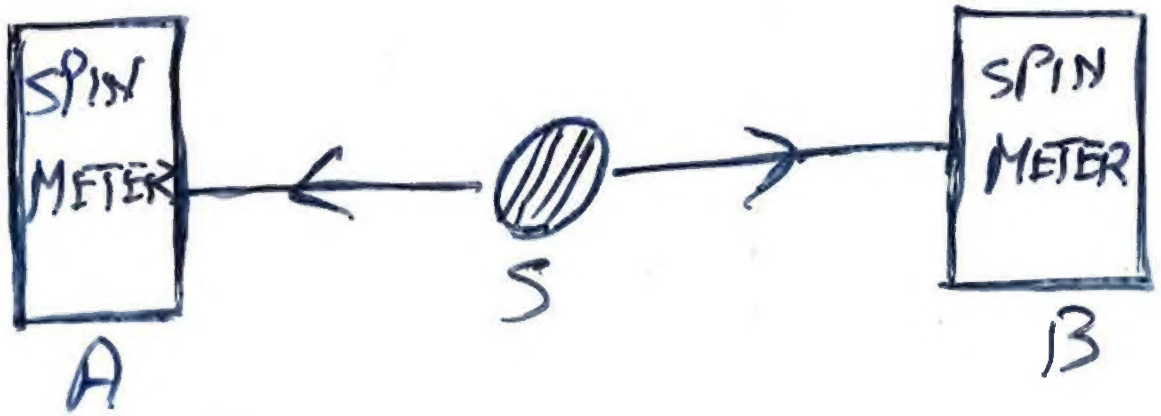
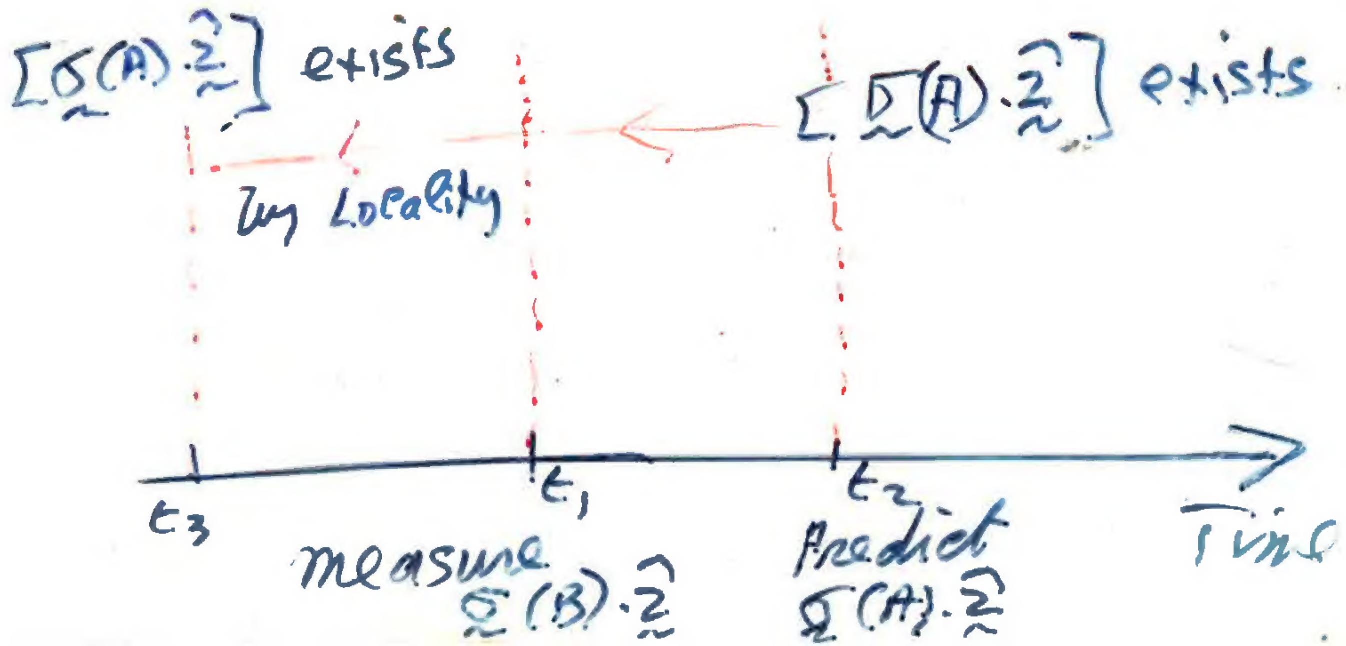
EPR ARGUMENT (1935)



$$|\Psi_{singlet}\rangle = \frac{1}{\sqrt{2}}\Big(|\underset{\sim}{\sigma}(A)\cdot\hat{\underset{\sim}{z}} = +1\rangle\, |\underset{\sim}{\sigma}(B)\cdot\hat{\underset{\sim}{z}} = -1\rangle - |\underset{\sim}{\sigma}(A)\cdot\hat{\underset{\sim}{z}} = -1\rangle\, |\underset{\sim}{\sigma}(B)\cdot\hat{\underset{\sim}{z}} = +1\rangle \Big)$$

(Mirror Image Correlations)



EINSTEIN DILEMMA

QM formalism

$\Rightarrow$ nonlocality OR incompleteness

$\downarrow$

completed version of QM (hidden variables)

$\downarrow$

Bell Inequalities

$\downarrow$ violated by Expt.

nonlocality

So QM is nonlocal simpliciter!

PROOFS OF THE BELL INEQUALITY ③

? + Locality + hidden variables
$\Rightarrow$ Bell Inequality

Bell (1964) assumes Determinism
plus Probability structure.
(e.g. J.P. for incompatible observables)

Two Controversies

① Does proof of Bell, under determinism, commit one to J.D.?

Fine (1982) says yes

Redhead (1983, 1988) and others say no
(Stapp (1971) Eberhard (1977) Svetlichny, Redhead, Brown and Butterfield (1988))

[Fine's work culminates in Pitowski (1989) work on generalized Bell inequalities as facet inequalities defining a multi-dimensional polytope]

(2) Can the Stapp–Eberhard proof be extended to cover indeterminism?

Stapp says <u>yes</u>

Redhead (& others) say <u>no</u>

[Hellman (1982) Redhead (1983, 1987) Clifton, Butterfield and Redhead (1990) — "A Stapp in the Wrong Direction!"].

SO, can a proof be given of the Bell inequality assuming indeterminism?

<u>yes</u> (Bell (1971)) but only assuming some probability structure.

↓

General line of this approach culminated in Jarrett (1984)

Can one give proofs of nonlocality which do not use probability at all?

This is the aim of <u>ALGEBRAIC PROOFS of NONLOCALITY</u>

HISTORY OF THE ALGEBRAIC PROOFS

(1) Project: Derive a Kochen-Specker (1967) contradiction for two spin-$\frac{1}{2}$ systems.

i.e. show local observable like $\underset{\sim}{\Sigma}(A) \cdot \hat{\underset{\sim}{z}}$ must depend on total context of properties of the *whole* system.

Proposed by Bub in 1976 in form of a question:
Could Maczynski's Theorem (1971) be shown *not* to be extendible from maximal to locally maximal observables?
But M's Theorem was so extended by Demopoulos, Humphreys and Bub in 1980
so no algebraic proof of *nonseparability* could be given.

(2) Heywood & Redhead (1983) derived a K-S contradiction on one of a pair of spin-1 particles, assuming Separability and Locality and Determinism. proof involved locally non-maximal observables

(3) Stairs (1983) followed by Brown and Svetlichny (1990) have given a similar type of proof but involving only locally maximal observables

(4) Elby (1990) produced a stochastic generalization of Stairs - Brown - Svetlichny

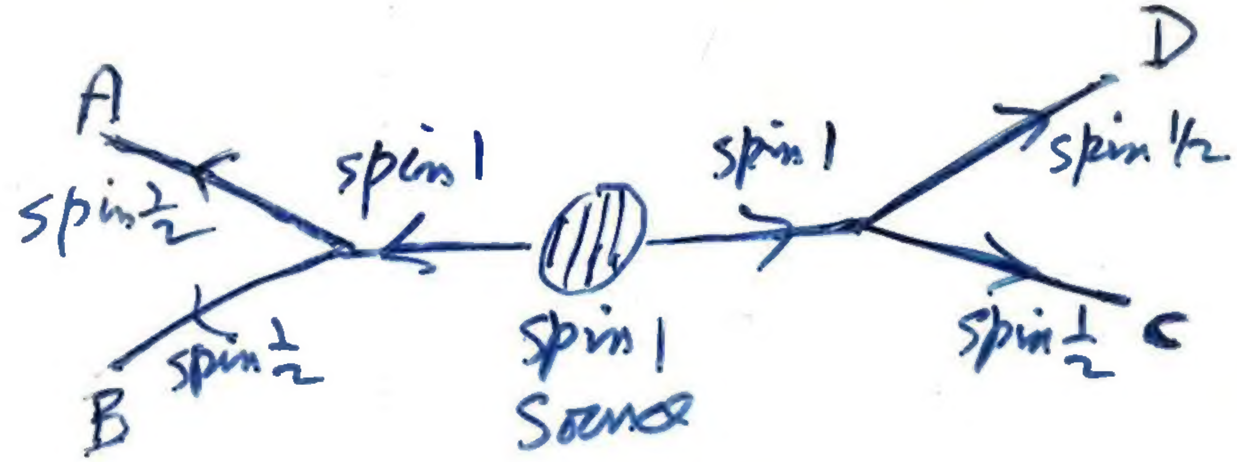
(5) Greenberger, Horne and Zeilinger (1989) produced a new deterministic "algebraic" proof of nonlocality, using correlations in a four-body decay.

(6) Redhead and Clifton (1990) showed that Greenberger's proof contains a flaw
→ new proof by Clifton that does work (!)

(7) Mermin (1990) showed connection between GHZ and K-S.

(8) Redhead and Pagonis (1991) generalized Mermin for arbitrary N and showed as $N \to \infty$, that Global Nonlocality fails for infinite systems.

GHZ proof



$$\theta_A + \theta_B - \theta_C - \theta_D = 0 \quad *$$

$$\Rightarrow A(\theta_A) \cdot B(\theta_B) \cdot C(\theta_C) \cdot D(\theta_D) = -1$$

Consider 4 possible settings for $\{\theta_A, \theta_B, \theta_C, \theta_D\}$ satisfying $*$

θ_A	$\rightarrow$	$\downarrow$	$\rightarrow$	$\uparrow$
θ_B	$\rightarrow$	$\uparrow$	$\uparrow$	$\rightarrow$
θ_C	$\rightarrow$	$\rightarrow$	$\uparrow$	$\uparrow$
θ_D	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$

Then we obtain

$$A(\rightarrow) B(\rightarrow) = A(\downarrow) B(\uparrow)$$
$$B(\rightarrow) C(\rightarrow) = B(\uparrow) C(\uparrow)$$
$$A(\rightarrow) C(\rightarrow) = A(\uparrow) C(\uparrow)$$

Multiplying and using $A^2 = B^2 = C^2 = 1$

gives $\quad 1 = A(\downarrow) A(\uparrow)$

Multiply by $A(\downarrow)$

$$\Rightarrow \quad A(\downarrow) = A(\uparrow)$$

But $A(\downarrow) = -A(\uparrow)$

$\therefore$ Contradiction

---

(7) Clifton has generalized GHZ proof to the stochastic case (similar in a general way to what Elby did for Stairs)

So where are we
left ?

Correlations that cannot
be explained

But the correlations are
not causal :

(1) No-Signalling theorems of
Ghirardi, Rimini & Weber
(1980) and others

(2) Non robustness of the
correlations
Redhead (1986)

Shimony – Passion-at-a-distance.

But we can actually prove $A(\downarrow) = -A(\uparrow)$

Thus note that, if $\theta_A + \theta_B - \theta_C - \theta_D = \pi$, it can be shown in the GHZ system that $A(\theta_A) \cdot B(\theta_B) \cdot C(\theta_C) \cdot D(\theta_D) = +1$

So choose

θ_A	$\uparrow$	$\downarrow$
θ_B	$\rightarrow$	$\rightarrow$
θ_C	$\uparrow$	$\uparrow$
θ_D	$\rightarrow$	$\rightarrow$

which yields

$$\left.\begin{array}{l} A(\uparrow) \cdot B(\rightarrow) \cdot C(\uparrow) \cdot D(\rightarrow) = -1 \\ A(\downarrow) \cdot B(\rightarrow) \cdot C(\uparrow) \cdot D(\rightarrow) = +1 \end{array}\right\}$$

whence it follows immediately that

$$A(\downarrow) = -A(\uparrow)$$

Generalized Mermin Proof

Consider N spin-$\frac{1}{2}$ particles

Define

$$T_1 = \sigma_x^1 \sigma_y^2 \sigma_y^3 \cdots \sigma_y^N$$

$$T_2 = \sigma_y^1 \sigma_x^2 \sigma_y^3 \cdots \sigma_y^N$$

$$\vdots$$

$$T_N = \sigma_y^1 \sigma_y^2 \sigma_y^3 \cdots \sigma_x^N$$

$$T_{N+1} = \sigma_x^1 \sigma_x^2 \sigma_x^3 \cdots \sigma_x^N$$

Then if $N = 1, 5, 9 \cdots$

$$T_1 T_2 T_3 \cdots T_N T_{N+1} = +1$$

while if $N = 3, 7, 11 \cdots$

$$T_1 T_2 T_3 \cdots T_N T_{N+1} = -1$$

But in all cases

$$[T_1 T_2 T_3 \cdots T_N T_{N+1}]$$

$$= ([\sigma_x^1])^2 \times ([\sigma_x^2])^2 \times \cdots$$

$$\times ([\sigma_y^1])^{N-1} \times ([\sigma_y^2])^{N-1} \times \cdots = +1$$

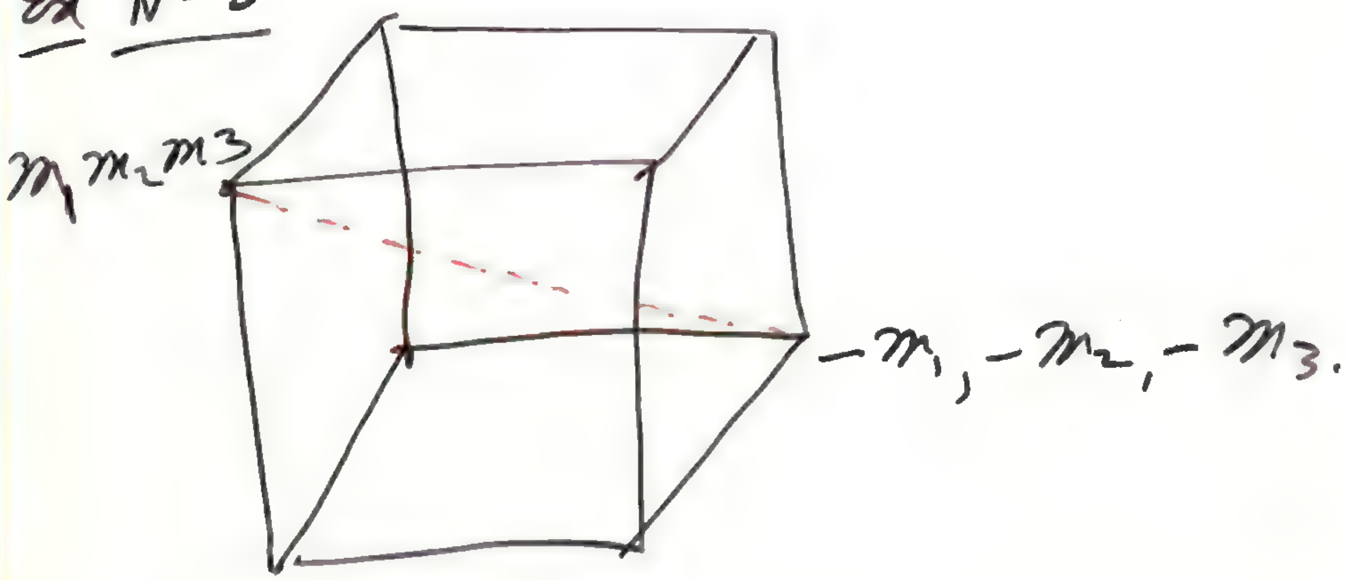
simultaneous eigenstates, for odd N, (14)
of $T_1 \cdots T_N$ are of form

$$\Psi = \frac{1}{\sqrt{2}}\left(|m_1 \cdots m_N\rangle \pm |-m_1 \cdots -m_N\rangle\right)$$

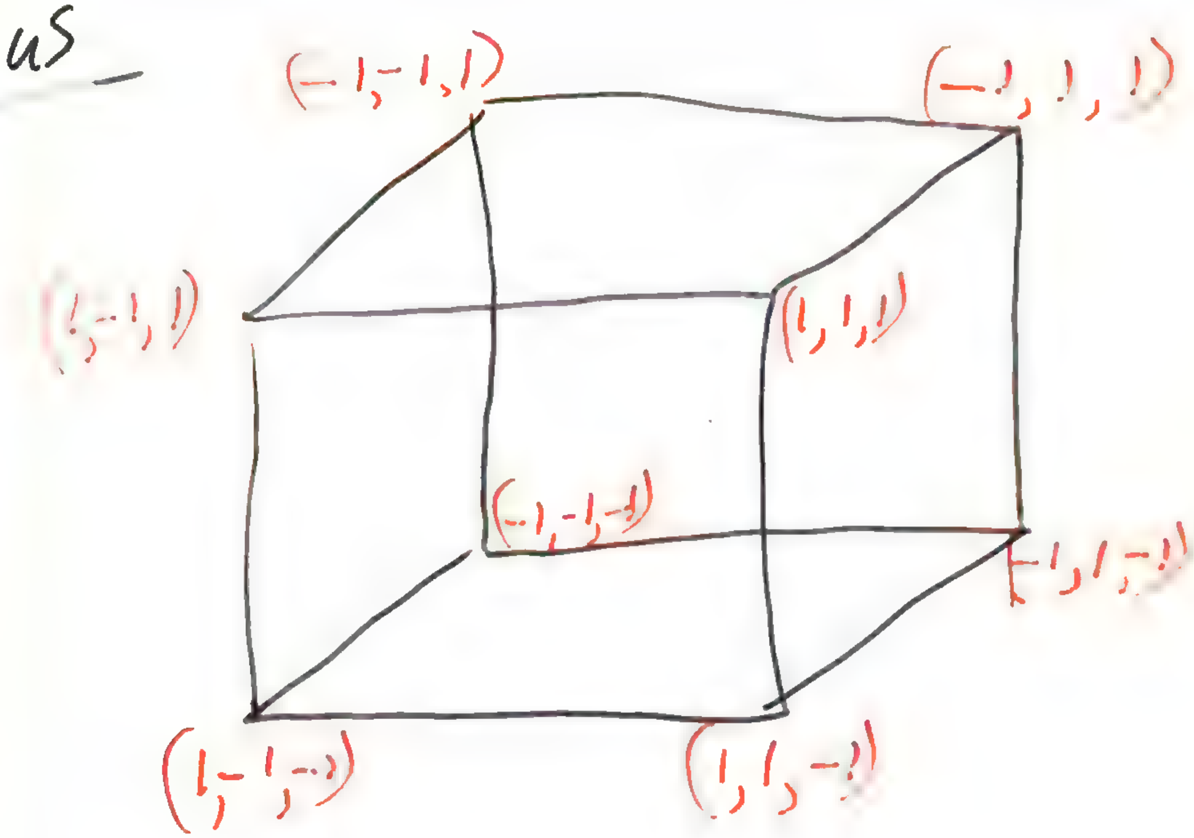
where $m_1 = \pm 1, \; \cdots \; m_N = \pm 1$

and $|m_1 \cdots m_N\rangle$ is simultaneous
eigenstate of $\sigma_z^1 \sigma_z^2 \cdots \sigma_z^N$.

Ex $N=3$



Thus



Now let $N \to \infty$
through values 3, 7, 11 - - - -

Does the infinite system
exhibit "global" nonlocality?

GLOBAL NONLOCALITY

The Mermin Table

$$
\boxed{\begin{array}{ccc}
x & y & y \\
y & x & y \\
y & y & x \\
x & x & x
\end{array}}
$$

$N = 3$

Generalized Mermin Table

$$
\boxed{\begin{array}{cccccc}
x & y & y & \cdots & & y \\
y & x & y & \cdots & & y \\
\vdots & & \ddots & & & \\
 & & & & \ddots & x \\
y & y & y & \cdots & & \\
 & & & & & x \\
x & x & x & \cdots & &
\end{array}}
$$

$N = 3, 7, 11 \ldots\ldots$

Odd $N \geqslant 3$

$$\left[\begin{array}{ccc|cccc}
x & y & y & y & y & \cdots & y \\
y & x & y & y & y & \cdots & y \\
\hline
y & y & x & x & x & \cdots & x \\
x & x & x & x & x & \cdots & x
\end{array}\right]$$

Even $N \geqslant 4$

$$\left[\begin{array}{c|ccc|cccc}
x & y & y & y & y & y & \cdots & y \\
x & y & y & y & y & y & \cdots & y \\
\cline{2-4}
y & x & y & y & y & y & \cdots & y \\
y & y & x & y & y & y & \cdots & y \\
\cline{2-8}
y & y & y & x & x & x & \cdots & x \\
y & x & x & x & x & x & \cdots & x
\end{array}\right]$$

Mermin Contradiction for arbitrary 3N $N \geq 1$

$x\ y\ y$	$x\ y\ y$	$\cdots$	$x\ y\ y$
$x\ y\ y$	$y\ x\ y$	$\cdots$	$y\ x\ y$
$x\ y\ y$	$y\ y\ x$	$\cdots$	$y\ y\ x$
$y\ x\ y$	$x\ y\ y$	$\cdots$	$x\ y\ y$
$y\ y\ x$	$y\ x\ y$	$\cdots$	$y\ x\ y$
$x\ x\ x$	$y\ y\ x$	$\cdots$	$y\ y\ x$

Mermin Proof of Nonlocality

(13a)

Consider 3 spin-$\frac{1}{2}$ particles

Define

$$T_1 = \sigma_x^1 \sigma_y^2 \sigma_y^3$$

$$T_2 = \sigma_y^1 \sigma_x^2 \sigma_y^3$$

$$T_3 = \sigma_y^1 \sigma_y^2 \sigma_x^3$$

$$T_4 = \sigma_x^1 \sigma_x^2 \sigma_x^3$$

Then $T_1 T_2 T_3 T_4 = \underline{-1}$

But

$$[T_1 T_2 T_3 T_4]$$

$$= ([\sigma_x^1])^2 \times ([\sigma_x^2])^2 \times ([\sigma_x^3])^2$$

$$\times ([\sigma_y^1])^2 \times ([\sigma_y^2])^2 \times ([\sigma_y^3])^2$$

$$\underline{= +1}$$

Kochen - Specker Contradiction

Denote possessed value of Q by $[Q]$

If. $A = f(c) = g(c')$

where c, c' are maximal & $[c, c'] \neq 0$

then A must be non-maximal.

Apply FUNC: $[f(Q)] = f([Q])$

So $[A] = \underline{f([c]) = g([c'])}$

This constraint on incompatible c, c' leads to K-S contradiction

Solution: Distinguish A_c & $A_{c'}$

where $[A_c] \underset{Df}{=} f([c])$

$[A_{c'}] \underset{Df}{=} g([c'])$

For two systems A & B $\qquad \dot{A} \qquad \dot{B}$

OLOC $\left[A_{(A,B)}\right] = \left[A_{(A,B')}\right] = [A]$

where A is locally maximal

ELOC $[A](A,B) = [A](A,B')$

Environmental context

Mermin and the Connection with the Kochen-Specker Paradox

$$T_1 = \sigma_x^1 \sigma_y^2 \sigma_y^3$$

$$T_2 = \sigma_y^1 \sigma_x^2 \sigma_y^3$$

$$T_3 = \sigma_y^1 \sigma_y^2 \sigma_x^3$$

$$T_4 = \sigma_x^1 \sigma_x^2 \sigma_x^3$$

$$T_1^2 = T_2^2 = T_3^2 = T_4^2 = 1$$

$$T_4 = - T_1 T_2 T_3$$

So $T_1 T_2 T_3 T_4 = -1$

Denote value of T_1 in the state ϕ in the context of the unique orthonormal basis which simultaneously diagonalizes T_1, T_2, T_3 by $[T_1]^{\phi}_{T_1 T_2 T_3}$

etc.

Similarly we define $[T_1]^{\phi}_{xyy}$ in the context which simultaneously diagonalizes

Then we have, for arbitrary ϕ

$$[T_1 T_2 T_3 T_4]^{\phi}_{T_1T_2T_3} = [-]^{\phi}_{T_1T_2T_3}$$

$$= -1$$

whence

$$[T_1]^{\phi}_{T_1T_2T_3} \times [T_2]^{\phi}_{T_1T_2T_3} \times [T_3]^{\phi}_{T_1T_2T_3}$$
$$\times [T_4]^{\phi}_{T_1T_2T_3} = -1$$

Also

$$[T_1]^{\phi}_{XYY} = [\sigma_x^1]^{\phi}_{XYY} \times [\sigma_y^2]^{\phi}_{XYY}$$
$$\times [\sigma_y^3]^{\phi}_{XYY}$$

etc.

If we have no contextualization we get the K–S contradiction

$$([\sigma_x^1]^{\phi})^2 \times ([\sigma_y^1]^{\phi})^2 \times ([\sigma_x^2]^{\phi})^2$$
$$\times ([\sigma_y^2]^{\phi})^2 \times ([\sigma_x^3]^{\phi})^2 \times ([\sigma_y^3]^{\phi})^2$$
$$= -1$$